ROBUST CONTROL OF A HEAT EXCHANGER USING A SMITH PREDICTOR

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abstract

This paper investigates the robustness of a time delayed system controlled by a PI controller with a Smith Predictor used to compensate the dead time. A heat exchanger process is used as a test bed for the proposed control structure. Feed-forward control is used to compensate for the disturbance effect. The effectiveness of the design procedure is tested using Matlab-Simulink models and frequency analysis for the robustness is performed.

Keywords: Smith Predictor, time-delay system, robust control

1. Introduction

Time delays between inputs and outputs are very common in industrial processes. Typical examples of time-delay systems are communication networks [4], chemical processes [13], tele-operation systems [1] etc. For processes with long time delays it is often difficult to achieve good control using just PID control strategies. The effect of the dead time on the process reduces the stability of the system and limits the achievable response time of the system.[3] By adding a Smith Predictor to the control loop it is possible to compensate the system’s time delay effects. The Smith Predictor control structure contains the process mathematical model in the feedback loop. However, the exact plant model and the delay time are needed in order for the Smith Predictor to have the desired effect [1,15].

Most of the time, for non-linear systems, such as the heat exchanger process considered in this paper, plant models are affected by uncertainties and cannot be modeled exactly. These processes may exhibit gain and time constant changes as well as varying time delays which need to be compensated robustly in order to maintain the closed loop specifications both under nominal conditions and under time delay uncertainties.[13]

In this paper, a Smith Predictor is designed for a heat exchanger process. The limitations of PI control as compared to the more effective Smith Predictor control strategy are illustrated by means of digital simulation. The robustness of the design to model uncertainties is analyzed in both time and frequency domain. Simulation results show also the tracking capabilities of the system as well as the disturbance rejection.

2. Smith Predictor

The general form of a time-delay SISO process is given by:

\[ P(s) = G(s)e^{-\tau s} \]  (1)

where \( G(s) \) is a delay free transfer function and \( \tau \) is the time delay.

The most common way to control such processes is to use the Smith Predictor. The structure of this controller is shown in fig. 1.

Fig. 1 – Smith Predictor control structure

The closed loop transfer function from \( y_{sp} \) to \( y \) is given as:

\[ T(s) = \frac{P(s)C(s)}{1 + P(s)C(s) + G_p(s)C(s) - G_p(s)C(s)e^{-\tau s}} \]  (2)

where \( G_p(s) \) is the nominal plant model and \( C(s) \) is the controller. If the Smith Predictor uses the same plant model as the delay free transfer function in \( P(s) = G(s) \), then transfer function (2) becomes:

\[ T(s) = \frac{P(s)C(s)}{1 + G_p(s)C(s)} = \frac{G(s)C(s)}{1 + G_p(s)C(s)} e^{-\tau s} \]  (3)

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This way, the closed-loop transfer function, except for the factor $e^{-\tau s}$, has the same form as the closed-loop system controlled by $C(s)$ in the absence of time delay. The resulting control structure provides a delayed response with the same shape as that of the delay-free system, as if the delay were shifted outside the feedback loop. [9,15]

3. The heat exchanger process

The heat exchanger shown in fig. 2 is considered. The top inlet delivers fluid to be mixed in the tank. To regulate the temperature of the tank fluid to a desired set point, the amount of steam supplied to the heat exchanger is modified via its control valve. Variations in the temperature of the inlet flow are the main source of disturbances in this process [11,14].

![Fig. 2 – Reactor with heat exchanger](image)

In [5,11] the heat exchanger model is identified. The considered plant model is typical of a first order plus dead-time (FOPDT) and is given by:

$$P(s) = \frac{k_f}{T_f s + 1} e^{-\tau s}$$  \hspace{1cm} (4)

where: $\tau = 14.7s$, $k_f = 1$ (normalized units were considered) and $T_f = 21.3s$.

The transfer function in (4) models the way a change in the voltage driving the steam valve opening affects the tank temperature.

The model of the inflow disturbance is shown in equation (5).

$$D(s) = \frac{1}{25s + 1} e^{-3s}$$  \hspace{1cm} (5)

The transfer function in (5) models the way a change in the inflow temperature affects the tank temperature.[11]

4. Design of the control structure

A. Design of the feedback PI controller

The main controller $C(s)$ in the Smith Predictor shown in fig. 1 can be chosen to be a proportional-integral (PI) controller of the form:

$$C(s) = K_C \left[ 1 + \frac{1}{T_i s} \right]$$  \hspace{1cm} (6)

Assuming that the desired closed-loop response for the delay-free plant should have a time constant $T_m$, the following controller design is obtained: [7,10]

$$T_i = T_f$$

$$K_C = \frac{T_f}{k_f T_m}$$  \hspace{1cm} (7)

According to [7], for the design of the feedback controller $C(s)$ in the Smith Predictor, the desired $T_m$ should be specified as a ratio of $T_m$ to the process time constant $T_p$, a suitable range of this ratio being 0.2 to 1.

The performance of the resulting PI controller is severely limited by the long dead time. Figure 3 shows the loss of stability when increasing the proportional gain $K_C$. While the closed-loop system presents a faster response time, the increase in overshoot quickly leads to instability.

![Fig. 3 – Loss of stability for feedback PI control when increasing $K_C$](image)

Figure 4 shows the compared response to a step input in case the system is controlled by the PI controller and for the Smith Predictor structure which considers the same controller. Disturbance rejection for the two designs is also shown in fig. 4.

The tuning parameters for $C(s)$ were computed based on (7) and considered $T_m=18s$.

![Fig. 4 – Compared step response for PI controller and Smith Predictor](image)
As seen in fig. 4 the Smith Predictor improves the system’s response to set point changes, but in the task of disturbance rejection both control structures show a similar behavior which could be improved.

B. Model mismatch

If there isn’t an exact match between the plant and the internal model used by the Smith Predictor, then it is important to study the robustness of the Smith Predictor to uncertainties in the process dynamics and dead time. Figure 4 showed the system’s response when the internal model \( G_p(s)e^{-\tau s} \) matched the plant model \( P(s) \) exactly. This is not the case in practical situations, when the internal model is only an approximation of the real plant.

Equation (4) defines a nominal model of the heat-exchanger process, characterized by the three parameters: \( k_f \), \( T_f \) and \( \tau \). The uncertain model of the process considers a variation of the process parameters within a specific range:

\[
\begin{align*}
\tilde{k}_f &= \left[ k_f + \Delta k_f \right] \\
\tilde{T}_f &= \left[ T_f + \Delta T_f \right] \\
\tilde{\tau} &= \left[ \tau \pm \Delta \tau \right]
\end{align*}
\] (8)

The values \( k_f \), \( T_f \) and \( \tau \) are the nominal values of the process parameters, and \( \Delta k_f \), \( \Delta T_f \) and \( \Delta \tau \) are the maximum deviations from the nominal values [6]. According to (4) and (8), there are many admissible transfer functions for this process, defined by each possible combination of \( k_f \), \( T_f \) and \( \tau \) in the given intervals. For the purpose of this study two representative plant models were considered within the specified range:

\[
\begin{align*}
\text{P}_1(s) &= \frac{1.2}{23s + 1} e^{-17s} \\
\text{P}_2(s) &= \frac{0.8}{18s + 1} e^{-12s}
\end{align*}
\] (9)

and the analysis of fig. 4 was performed again. The time response to a step change in the set point and disturbance are shown in fig. 5.

As seen in these figures, both designs are sensitive to model uncertainties. However, the Smith Predictor control structure seems to outperform the simple PI controller for the task of set point tracking. In the case of disturbance rejection a better response is needed.

To test the Smith Predictor’s sensitivity to modeling errors a frequency analysis of the open loop form \( y_{sp} \) to \( d \) is performed in order to check the stability margins. The perturbed process models are used for this analysis. The Bode diagram for \( P_1(s) \) is shown in fig. 6.

The magnitude curves show a gain margin of 16dB for \( P_1 \) and \( P_2 \), while the phase margin is infinite. According to [8,12] these values assure the closed loop stability and robustness of the system to model uncertainties. However, as seen in fig. 5 better disturbance rejection is needed.

C. Design of the feed-forward controller

To improve disturbance rejection, a feed-forward controller is added to the control structure in fig.1. The modified structure of the system with the feed-forward controller, and Smith Predictor is shown in fig. 7.

\[
y = (P \cdot F + D)d
\] (10)

According to [2, 11], in order to eliminate the effect of the measured disturbance we need to choose the feed-forward controller \( F \) so that:
This leads to:

\[ F(s) = \frac{D(s)}{P(s)} = \frac{21.3s + 1}{25s + 1} e^{-25s} \]  \hspace{1cm} (12)

In reality, modeling inaccuracies prevent exact disturbance rejection, but feed-forward control will help minimize temperature fluctuations due to inflow disturbances. [11]

Figures 8 and 9 show the compared response to a step change in the set-point and for a step change in the disturbance for each of the following three cases:
- case 1: the system is controlled by the PI + feed-forward controller (fig. 8);
- case 2: the system is controlled by the Smith Predictor structure + feed-forward controller (fig. 9);
- case 3: the system is controlled by a PI controller designed using Ziegler-Nichols tuning method, with the parameters of equation (13) + feed-forward controller (fig. 10):

\[ K_c = \frac{0.9T_f}{\tau} = 1.3 \]
\[ T_f = 3.3\tau = 48.5 \]  \hspace{1cm} (13)

In all cases the nominal \( P(s) \) and perturbed plants \( P_1(s) \) and \( P_2(s) \) are considered.

By analyzing the time response of the three control structures, the following performances for set-point tracking are summarized in table 1.

<table>
<thead>
<tr>
<th>Table 1. Performances</th>
<th>SP</th>
<th>PI</th>
<th>PI (Z-N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overshoot</td>
<td>0</td>
<td>31.8%</td>
<td>1.82%</td>
</tr>
<tr>
<td>Settling time</td>
<td>84.9s</td>
<td>135s</td>
<td>206s</td>
</tr>
</tbody>
</table>

Figure 11 shows the controlled output in the task of set point tracking for the nominal plant. The set-point changes in steps at time 50s, 350s and 650s. A step disturbance \( d = -0.5 \) is applied to the system at time \( t = 150s \). The system is controlled by the Smith Predictor structure + feed-forward controller of fig. 7.

5. Conclusion

In this paper the robustness of a Smith Predictor control structure for a heat exchanger process is analyzed. When compared to the classical PI, the Smith Predictor greatly improves the system’s response to set-point changes. When compared to the performances of a PI controller designed using Ziegler Nichols, the closed loop system shows a substantial decreases in settling time. While the designed Smith Predictor seems to perform robustly to changes in the set-point and to model uncertainties, the disturbance needs to be compensated separately.
using a feed-forward controller. The introduction of the feed-forward controller reduces the amplitude of the disturbance effect on the output signal $y$ better than it was possible under the control of the Smith predictor alone. The proposed control structure of fig. 7 combines the benefits of the Smith predictor regarding robustness to model uncertainties and set-point changes and that of feed-forward control for disturbance rejection.

References


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